Pricing Ramping-Constrained Multi-Period Economic Dispatch

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Introduction

- With renewable energies, the duck curve is real

- Greentech media report: CAISO 2015, the 3-hour ramp > 5000 MW over 58% of the time (only 6% in 2011)
- Ramping-constrained multi-period dispatch, correct price signals
Introduction

• A 2-gen example (From CAISO report)

Load: L1=420, L2=590

\[
\begin{array}{cccc}
\text{Capacity} & \text{Marginal Price} & \text{Ramp} \\
G1 & 500 & 25 & 500 \\
G2 & 500 & 30 & 50 \\
\end{array}
\]

\[
\begin{array}{cccc}
\text{Dispatch/} & \text{LMP t=1} & \text{Dispatch/} & \text{LMP t=2} \\
\text{Load} & (420, 25) & \text{Load} & (590, 35) \\
G1 & (380, 25) & G1 & (500, 35) \\
G2 & (40, 25) & G2 & (90, 35) \\
\end{array}
\]

- G2 is underpaid at t=1, expects to be reimbursed at t=2
- However, such expectations may not become real
Introduction

• Rolling-window dispatch
  • G2 (price 30) would rather quit the market
  • (Elastic) Load would consume more
  • LMPs do not constitute market equilibrium

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<tr>
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<th>Dispatch/LMP t=1</th>
<th>Dispatch/LMP t=2</th>
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<th>Dispatch/LMP t=3</th>
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<tbody>
<tr>
<td>G1</td>
<td>(380, 25)</td>
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<td>(500, 30)</td>
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</tr>
<tr>
<td>G2</td>
<td>(40, 25)</td>
<td>(90, 35)</td>
<td>(90, 30)</td>
<td>(90, 30)</td>
</tr>
<tr>
<td>Load</td>
<td>(420, 25)</td>
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<td>(590, 30)</td>
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</tbody>
</table>

Load: L1=420, L2=590, L3=590, …
Introduction

- Flexible Ramping Product
  - Treating ramping as a public good that are uniformly priced.
  - Ramping prices are paid to generators and charged to demands.

\[
\begin{align*}
\text{FRP:} & \quad \text{minimize} \quad \sum_{i,t} f_{it}(P_{it}) \\
\text{subject to} & \quad \sum_i P_{it} = L_t, \quad (\lambda_t) \\
& \quad \sum_i R_{it}^+ = \Delta_t, \quad (\xi_t) \\
& \quad P_{it} + R_{it}^+ \leq \bar{P}_{it}, \\
& \quad 0 \leq R_{it}^+ \leq \bar{R}_{it}, \\
& \quad P_{i(t+1)} - P_{it} \leq \bar{R}_{it}.
\end{align*}
\]

- An ad hoc way, may have the same problem, e.g., there is a unit with a high price & high ramping rate
- CAISO imposes a cap on energy prices for ramping sources

- Generation payment: \( \lambda_t P_{it} + \xi_t R_{it}^+ \)
- Demand payment: \( (\lambda_t - \xi_t + \xi_{t-1})L_t \)
Outline

• Single-period economic dispatch and pricing

• Multi-period economic dispatch and pricing
  • One-shot solution vs. Rolling-window dispatch
  • LMP vs. Temporal locational marginal price (TLMP)
  • General equilibrium vs. Partial equilibrium

• Ex post pricing
Single-period problem

- Bid-based economic dispatch
  - Participants submit bids and offers
  - The system operator
    - clears the market by setting the generation/consumption levels
    - settles the market by setting prices in ex ante or ex post.

- Economic dispatch

- LMP: Same for generators & load

\[
\pi = \frac{d}{dL} C(L) = \lambda^*
\]

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Theorem (F. Wu, P. Varaiya, etc.) The pair (optimal dispatch \( P_i^* \), LMP \( \lambda^* \)) defines an efficient competitive equilibrium

- Market clear: Total supply = Total demand
- Individual rationality:
  \[ P_i^* = \operatorname{argmax}_p (\lambda^* p - f_i(p)) \]
- (Efficiency): \( P_i^* \) optimally solves the ED

- Assume gens are price takers, they will voluntarily follow dispatch instructions

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One-shot Multi-period ED

\[ P_1: \text{minimize} \sum_{i,t} f_{it}(P_{it}) \]

subject to

\[ \sum_i P_{it} = L_t, \]

\[ R_{it} \leq P_{i(t+1)} - P_{it} \leq \bar{R}_{it}, \quad (\mu_t, \bar{\mu}_t) \]

\[ \frac{\partial}{\partial L_t} C(L) = \lambda_t \]

LMP: generator and load are priced at

General (Competitive) equilibrium

- Electricity at different time (and location) are treated as different goods: \( \{(\pi_t, p_{it}), t = 1, \cdots, T\} \).
- Each market participant is rational:

\[ (p_{it}^*) = \arg \max _{(p_t) \in P_t} \sum_i (\pi_t p_t - f_{it}(p_t)) \]

- Market clears: \( (p_{it}^*) \in N(L) \).

Theorem. LMP is an efficient general equilibrium price.
Partial equilibrium

- Electricity at different time (and location) are treated as different goods: \( \{(\pi_t, p_{it}), t = 1, \ldots, T\} \).
- Each market participant is rational at time \( t \):
  
  \[
  (p^*_{it}) = \arg \max_{(p_t) \in P_i} \left( \pi_t p_t - f_i(p_t) \right) \quad \forall i.
  \]
- Market clears at \( t \): \( (p^*_{it}) \in N(L_t) \).

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**Theorem.** When some of the ramping conditions are binding, in general, no partial equilibrium (uniform) price exists for the multi-period economic dispatch

(i) out-of-market settlement or (ii) discriminative pricing
Temporal Locational Marginal Price

\[ P_1: \text{minimize } \sum_{i,t} f_{it}(P_{it}) \]

subject to

\[ \sum_i P_{it} = L_t, \]

\[ R_{it} \leq P_{i(t+1)} - P_{it} \leq \bar{R}_{it}, \quad (\lambda_t) \]

\[ \mu_{i(t-1)} - \mu_{i(t-1)} = \lambda_t + \Delta_{it} \]

\[ C_{jt}: \text{minimize } \sum_{(i,t) \neq (j,t)} f_{it}(P_{it}) \]

subject to

\[ \sum_i P_{it} = L_t, \]

\[ R_i \leq P_{i(t+1)} - P_{it} \leq \bar{R}_i, \quad (\lambda_t) \]

\[ (\mu_{it}, \bar{\mu}_{it}) \]

Ramping component

- Take \( P_{it} \) for a parameter
- Represents the marginal benefit (cost) of all the other market participants wrt. a small change of \( P_{it} \)
- Overall willingness to buy from \( P_{it} \) aggregated demand curve
Temporal Locational Marginal Price

Theorem (Efficient market equilibrium). The TLMP $\pi^{\text{TLMP}}$ and the multi-period economic dispatch $P^*$ satisfy

- **Optimal market clearing;**

- **Individual rationality:**
  \[ P_{it}^* = \arg\max_{(p)\in\mathcal{P}_{it}} \left( \pi_{p_{it}}^{\text{TLMP}} p - f_i(p) \right), \quad \forall i, t \]

- **Revenue adequacy:**
  \[ \sum_t \pi_{L_t}^{\text{TLMP}} L_t \geq \sum_{it} \pi_{p_{it}}^{\text{TLMP}} P_{it}^*. \]
### TLMP vs. LMP One-Shot

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<th>Cost</th>
<th>Ramp</th>
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<tr>
<td>G1</td>
<td>500</td>
<td>25</td>
<td>500</td>
</tr>
<tr>
<td>G2</td>
<td>500</td>
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Load: \( L_1 = 420, \ L_2 = 590 \)

<table>
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<tr>
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<th>Dispatch/ LMP t=1</th>
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<th>Profit</th>
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<tr>
<td>G1</td>
<td>(380, 25)</td>
<td>(500, 35)</td>
<td>5000</td>
</tr>
<tr>
<td>G2</td>
<td>(40, 25)</td>
<td>(90, 35)</td>
<td>250</td>
</tr>
<tr>
<td>L</td>
<td>(420, 25)</td>
<td>(590, 35)</td>
<td>0 (SO)</td>
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- 250 profit shift from G2 to SO
- G2 has no incentive to improve its ramping with LMP
- No such a problem with TLMP
- Merchandise surplus similar to congestion

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<td>0</td>
</tr>
<tr>
<td>L</td>
<td>(420, 25)</td>
<td>(590, 35)</td>
<td>250</td>
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Merchandising surplus = 250
= \((\text{ramping price } 5) \times \text{(ramping rate } 50)\)
TLMP vs. LMP Rolling-window

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<td>500</td>
<td>30</td>
<td>50</td>
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Load: L1=420, L2=590

<table>
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- TLMP revenue inadequate
- LMP cannot adequately support G2 --- ultimately revenue inadequate
- Missing information, SO cannot obtain optimal primal & dual variables

Merchandising surplus = 250
= (ramping price 5) × (ramping rate 50)
Ex post dispatch & pricing

• A posteriori assessment with all information available based on the realized dispatch
  • Missing information in the rolling-window dispatch
  • Uncertainties, Gens deviating from their instructions, etc.

• The standard form: Incremental dispatch

minimize subject to

\[ \sum_{i,t} c_{it} \Delta P_{it} \]
\[ \sum_i \Delta P_{it} = 0, \]

and for all \( i \) and \( t = 1, \ldots, T \)

\[ l_i \leq P_{i(t+1)} + \Delta P_{i(t+1)} \]
\[-(P_{it} + \Delta P_{it}) \leq \bar{r}_i, \]
\[ 0 \leq P_{it} + \Delta P_{it} \leq \bar{P}_i, \]
\[-B_i \leq \Delta P_{it} \leq \bar{B}_i, \]

\[ (\lambda_t) \]
\[ (\mu_{it}, \bar{\mu}_{it}) \]
\[ (\bar{\mu}_{it}, \bar{\mu}_{it}) \]
\[ (\bar{\mu}_{it}, \bar{\mu}_{it}) \]

• Artificial lower & upper bounds \( \bar{B}_i \) and \( \bar{B}_i \)

• Used to be common, gradually lost to ex ante
Ex post incremental TLMP (iTLMPP)

minimize \[ \sum_{i,t} c_{it} \Delta P_{it} \]
subject to \[ \sum_{i} \Delta P_{it} = 0, \] \[ (\lambda_{t}) \]
and for all \( i \) and \( t = 1, \ldots T \)
\[ r_{i} \leq P_{i(t+1)} + \Delta P_{i(t+1)} \]
\[ -(P_{it} + \Delta P_{it}) \leq \bar{r}_{i}, \]
\[ 0 \leq P_{it} + \Delta P_{it} \leq \bar{P}_{it}, \]
\[ -B_{i} \leq \Delta P_{it} \leq B_{i}, \]
\[ (\mu_{it}, \bar{\mu}_{it}) \]
\[ (\rho_{it}, \bar{\rho}_{it}) \]
\[ (\eta_{it}, \bar{\eta}_{it}) \]

TLMP: \[ \pi_{it} = \nabla_{\Delta P_{it}} c_{it} = \lambda_{t} - \bar{\mu}_{it} + \mu_{it} + \bar{\mu}_{i(t-1)} - \mu_{i(t-1)} = \lambda_{t} + \Delta_{it} \]

- Set \( B_{i} = r_{i}/2 \) and \( \overline{B}_{i} = \bar{r}_{i}/2 \)
- Gens will gradually adjust to optimal (iTLMPP—partial equilibrium)
Ex post incremental TLMP (iTLMP)

- The iTLMP with $B_i = r_i/2$ and $\bar{B}_i = \bar{r}_i/2$ has the following properties
  - **Revenue adequacy of SO**: The SO always has non-negative net revenue
  - **Price support of Gens**: Generators who deviate less than $r/2$ from their non-zero optimal levels will be adequately supported
  - **Partial equilibrium**: Given iTLMP, generators maximize their profit at the optimal dispatch
    \[
    P_{it}^* = \arg \max_{(p) \in P_{it}} \left( \pi_{p_{it}}^\text{TLMP} p - f_i(p) \right), \ \forall i, t
    \]
  - **Incentive issue**: Higher ramping rate means a bigger chance to participate in the price determination process
Simulations

- 1000 random load sequences
- Rolling window dispatch
- Three pricing schemes: LMP, FRP, and Incremental TLMP

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</tr>
<tr>
<td>G2</td>
<td>500</td>
<td>30</td>
<td>50</td>
</tr>
<tr>
<td>G3</td>
<td>500</td>
<td>50</td>
<td>500</td>
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<tr>
<th></th>
<th>Revenue Adequacy</th>
<th>Gen price Support</th>
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<tbody>
<tr>
<td>LMP</td>
<td>100%</td>
<td>81.1%</td>
</tr>
<tr>
<td>FRP</td>
<td>100%</td>
<td>92.8%</td>
</tr>
<tr>
<td>i-TLMP</td>
<td>100%</td>
<td>99.2%</td>
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Conclusions

• Pricing multi-period dispatch is challenging, incomplete information is used by the operator & market participants.

• Information structures used by the operator and participants in scheduling and pricing decisions are often different.

• Tradeoffs have to be made among achieving efficiency, incentive compatibility, and complexity.

• TLMP (iTLMP) is discriminative (sin!), but it offers some interesting and worthy properties….
Thank you!

Slides available at my website
https://sites.google.com/site/yeguo2008/home